



Optimization of Distributed Energy Systems Through a Linear Programming Framework

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Abstract

Transitioning to sustainable energy systems requires new tools to face new challenges in energy security, climate change mitigation and socio-economic equity. The research at hand extends the foundation established by Rabe et al. (2022) to optimize distributed energy systems based on linear programming. In this work, we extend the model with scenario-based policy analysis, group-based economic fairness, and carbon penalties. Scenario analysis assesses the effects of dynamic policy changes like stricter renewable energy mandates and carbon pricing, on the energy mix and costs. Economic fairness requires energy resources to be distributed equitably, while carbon penalties internalize the environmental costs of carbon-intensive processes, thus incentivizing decarbonization. The results reveal a best-suited energy mix, achieving renewable energy targets with minimum costs, while effectively performing under changing policy constraints to provide a reliable grid. This framework aims to synthesize the myriad of insights from policy design literature on energy, allowing policymakers and other stakeholders to develop actionable roadmaps in support of sustainable energy transition.

Keywords: Distributed energy systems, Linear programming, Scenario analysis, Carbon penalties, Renewable energy, Energy policy.

1. Introduction

The shift to sustainable energy systems is one of the most pressing global challenges informed by the dual needs of energy security and climate change avoidance. We see great potential in distributed energy systems, which take the form of microgrids, grid-connected renewable energy sources, and the like, to exploit the benefits of energy efficiency, greenhouse gas emission reductions, and the resilience of energy supply [1]. In doing so, linear programming models became a key tool for energy system optimization, balancing different goals like cost, emissions and resource availability. The first work [1] established a mathematical model which unified renewable and non-renewable energy sources for distributed energy optimization. However, this model, while being broad, is too simplistic when considering the complexities of evolving policy frameworks, equity and the external costs of carbon emissions.

This study builds upon the original model by tackling four key dimensions. First, to characterize the impacts of dynamic policy changes such as more stringent renewable energy standards and carbon pricing on the efficient energy mix, we introduce a scenario-based policy analysis [2]. Scenario analysis provides a way to navigate uncertainty, explore alternative futures, and empower policymakers to craft more robust energy policies [3], [4], [5]. Second,

group-based economic fairness is integrated as a participatory phase in the equitable allocation of energy between renewable and non-renewable resources. This promotes social equity by ensuring that the economic costs and benefits are fairly distributed across energy producers and consumers [6], [7]. Thirdly, the model embeds carbon penalties for non-renewable energy sources to help internalize environmental costs, drive decarbonization and allow the energy system to be consistent with global climate targets [8]. Finally, a sensitivity analysis is performed to evaluate how the model responds to key parameters, such as carbon penalties, renewable energy mandates, and production cost coefficients. It also provides deeper insights into the trade-offs and dynamics within the energy system.

Furthermore, scenario-based policy analysis enables the exploration of the effects of potential policy changes on the energy sector, providing a useful framework for assessing trade-offs and synergies across multiple objectives [9]. More complex energy sector transformations, such as stricter renewable energy targets and new carbon penalties, change the energy mix toward low-carbon (renewable) technologies but may also increase costs or affect the availability of required resources [10], [11]. Adding a group-based approach to economic fairness to the model integrates another socio-economic aspect, making sure that the opportunities from renewable energy expansion are shared and not unevenly falling on different groups [12]. Moreover, carbon penalties provide an economic signal to reduce emissions from fossil fuels and increase investments in cleaner technologies [13], [14], [15]. The sensitivity analysis shows the robustness and adaptability of the model, while also establishing the most important parameters influencing cost efficiency, energy mix, as well as policy efficiency and effectiveness.

The proposed extensions address these gaps and thus help to advance decision-making in distributed energy systems and their relevance to current global challenges. Furthermore, this research complements the model's ability to support energy policy by offering a comprehensive framework that includes economic, environmental and social aspects [16]. Additionally, it provides actionable recommendations for stakeholders including policymakers, energy producers, and consumers who aim to navigate the complexities surrounding the energy transition [17], [18]. The implications of this research are relevant to the increasing consideration of decentralized energy technology as part of sustainable development goals and offer insights into how to integrate multi-dimensional considerations to promote an equitable, efficient, and sustainable energy future.

Research Gaps and Contributions

The research identifies the following significant research gaps:

- Limited incorporation of policy-driven dynamics, such as renewable mandates and carbon pricing, in previous models.
- Absence of socio-economic equity considerations in resource allocation within distributed energy systems.
- Lack of detailed sensitivity analysis to understand parameter-driven trade-offs in energy system performance.

Contributions of This Study

- Integration of scenario-based policy analysis to evaluate dynamic policy impacts on energy systems.
- Inclusion of group-based economic fairness to address socio-economic disparities.
- Incorporation of carbon penalties to internalize environmental costs and incentivize decarbonization.
- Conducting sensitivity analysis to assess the impact of key parameters on cost efficiency and energy mix, providing actionable insights for policymakers.

2. Literature Review

Energy optimization modelling is applied across a broad range of application areas such as achieving an optimal balance between energy supply and consumption, enhancing resource allocation, and minimizing production costs. Linear programming (LP) has become one of the mainstream technologies in energy system optimization thanks to its efficiency in solving large-scale and multi-variable problems [19], [20]. Due to their flexibility and ability to provide optimal solutions, linear programming (LP) models have been widely used in the context of renewable energy systems, helping planners to realize energy mixes that adhere sustainability objectives whilst minimizing costs [21]. The conventional linear programming (LP) approach can be enhanced by employing multi-objective optimization techniques, which tackle multiple, often conflicting goals, such as minimizing costs while reducing greenhouse gas emissions within distributed energy systems [22]. These capabilities give analysts insights into the economic and environment trade-offs related to different energy planning strategies. However, many optimization models assume static input parameters, making them rigid and less adaptable to changing policy contexts.

Integrating policy-driven constraints into energy optimization models has gained critical importance in promoting sustainable energy transitions. This involves aligning energy generation outputs with policy objectives by incorporating renewable energy share targets, carbon emission limits, and government subsidies into the optimization framework [23]. For instance, [24] explored the effect of renewable energy objectives in the European Union on energy allocation, finding that more ambitious renewable energy targets can markedly affect cost-efficient energy allocation. [25] investigated how subsidies for renewable energy sources, particularly solar and wind power, act as powerful drivers for accelerating the transition to greener energy systems. Their analysis highlighted that financial support is a cornerstone of renewable energy adoption. However, a significant limitation emerged: many existing models, while adept at integrating policy goals, lack the ability to adapt to the fast-changing nature of energy regulations. For instance, policy targets and financial incentives often shift based on new priorities or external pressures. This inability to simulate dynamic or uncertain policy contexts underscores a clear need for more flexible modelling approaches. Future models must not only incorporate current regulatory objectives but also explore a broad range of evolving scenarios to effectively guide sustainable energy transitions.

Analyzing scenarios has emerged as a vital approach for understanding how different policy conditions affect energy systems. By simulating hypothetical cases, researchers gain insights into the consequences of policy changes, such as renewable energy mandates, carbon pricing mechanisms, or variations in subsidy levels. This method allows for a detailed evaluation of their influence on the energy mix, associated costs, and emission levels [26], [27]. For instance, [28] performed scenario-based sensitivity analyses to explore the impact of carbon taxes on energy expense, revealing substantial sensitivity based on price point. Monte Carlo simulations have also been used to introduce uncertainty into policy evaluations, including variability in renewable energy output and market conditions [29]. Scenario analysis is a versatile approach to structuring economic-environmental trade-offs, and thus a potent tool for policymakers. Yet even though it has undoubted utility, few if any models incorporate scenario analysis in a systematic way into energy optimization frameworks, especially in terms of policy adaptability.

Despite significant advancements in energy modelling, many frameworks still struggle to adapt to the complexities of dynamic policy environments. Existing models often depend on static assumptions, limiting their capacity to respond to rapid changes in policy, such as sudden adjustments to renewable energy targets or modifications in subsidy structures [30], [31]. Although scenario analysis has been employed in certain contexts, it is often treated as a supplementary approach rather than being fully embedded within the optimization process

[32]. Moreover, limited attention has been given to the combined impacts of multiple policy instruments, such as the interactions between carbon taxes and renewable energy subsidies, which play a pivotal role in practical energy planning [33], [34]. These challenges highlight the pressing need for more comprehensive modelling approaches that embed scenario analysis within the optimization process. Such frameworks would allow for dynamic adaptation to a wide range of evolving policy conditions. The current study, representing a significant step forward in overcoming the major limitations of classical energy optimization models, offers a scenario-analysis integrated approach within the framework. In contrast to traditional static models, which assume fixed policy parameters, the model adopted here explores the effects of varying renewable energy targets, carbon limits, and subsidy policies on the final solution. In one modelled scenario, for example, higher shares of renewable energy are at the forefront only if penalties on carbon use are imposed, while the second scenario departs from lenient renewable mandates if new subsidies are offered for cleaner technologies. These scenarios are not academic exercises; they engage real-world decision-making, where key players now face a constant trade-off between financial viability and urgent environmental goals. Through this flexibility, the model offers policymakers a powerful instrument that connects the static assumptions of traditional models with the dynamic nature of real-world energy systems, providing the potential for more responsive and robust energy planning.

The inclusion of scenario analysis in energy optimization models can provide a flexible and policy-oriented framework for energy planning. This extension allows for the assessment of a range of policy scenarios, overcoming the limitations of static models while reflecting on trade-offs between economic and environmental goals. By incorporating stochastic components which accounts for the uncertainty of energy production and demand, the framework may be further expanded in future research. In addition, this approach could also be used in various regions and regulatory environments, providing useful insights to global energy transitions. Such flexibility, therefore, helps address the growing need to make energy models more flexible and adaptive to variations in policy.

3. Research Gap and Model Formulation

The study by [1] created a linear programming model to optimize distributed energy systems (DES) with special reference to the transition in Poland from a centralized, coal-based energy network towards a decentralized, renewable energy network. This model helps improve the allocation of resources and reduce reliance on fossil fuels while minimizing costs and ensuring energy security. However, previous works have only focused on static optimization parameters, ignoring aspects, such as the energy mix and system sustainability, driven by policy.

In actual energy systems, production decisions and economic outcomes are heavily influenced by policy changes like renewable energy mandates, carbon penalties, or clean energy subsidies. These regulations create dynamic trade-offs among environmental sustainability, economic efficiency, and grid reliability. Neglecting these factors risks developing solutions that may be cost-optimal in the short term but misaligned with longer-term policy and climate goals.

This study fills those gaps by extending the base model to include scenario-based policy analysis. The extended model captures different policy scenarios that lead to stricter renewable energy targets, higher carbon penalties or higher subsidies for renewable sources, and assesses their effect on the energy mix, total costs of production and sustainability. This allows for a more comprehensive framework within which policymakers and stakeholders can analyze trade-offs across a variety of regulatory settings, thereby improving the relevance of the model to the needs of different stakeholders. This study will have an impact on the design of adaptive strategies to meet energy demand in line with climate goals, ensuring energy security while maintaining economic viability in a changing environment.

3.1 Mathematical Model

The Objective function of the energy production optimization model contains multiple cost components associated with decision variables (to), which incorporates production costs, ecological certificate costs, EUA entitlement costs and energy storage costs for each type of energy. It also includes the cost of losing the fertility of the soil (to), which is the unit of effort to recover soil conditions after raw materials have been used for biomass, biogas, biofuels, and commodity agricultural production. These coefficients measure the ecological consequences of energy crop cultivation as one of the priorities in the optimization study. Table 1 details the decision variables included in the model [1].

Table 1. Decision variables used in the model.

Variable	Variable Name	Units
	Volume of energy derived from conventional non-renewable sources	kWh
	Electricity generated through co-incineration processes	kWh
	Hydropower contribution from existing installations	kWh
	Electricity output from new hydropower installations	kWh
	Total solar energy harnessed	kWh
	Wind energy produced by residential windmills	kWh
	Energy output from wind farms	kWh
	Additional energy sourced from wind power systems	kWh
	Energy derived from biogas systems established before the study	kWh
	Electricity produced via biogas in advanced cogeneration units	kWh
	Biogas-based energy output	kWh
	Energy contribution from biofuel production processes	kWh
	Biomass energy from boilers operating before recent upgrades	kWh

	Biomass energy generated in newly commissioned boiler systems	kWh
	Geothermal energy harvested	kWh
	Aggregate energy production from all sources	kWh
	Area cultivated for miscanthus, contributing to biomass energy production	ha
	Poplar cultivation area allocated for biomass energy	ha
	Land allocated for Slazowiec biomass production	ha
	Jerusalem artichoke cultivation supporting biomass energy generation	ha
	Rapeseed cultivation for biomass energy purposes	ha
	Cereal crops dedicated to biofuel energy generation	ha
	Corn grown for biomass energy production	ha
	Beet cultivation area utilized for biomass-related energy output	ha

The objective function for this optimization model is expressed as:

Equation (1) The objective function minimizes total production costs, incorporating both energy generation expenses and carbon penalties for non-renewable sources.

(2)

Equation (2) Total energy generated from renewable and non-renewable sources equals the required system energy volume.

Equation (3) The model enforces a minimum energy production threshold, allowing excess energy to be exported to neighboring regions.

(4)

Equation (4) emphasizes that at least half of the total energy generation must involve renewables

Equation (5) establishes a base threshold value of wind energy production in order to achieve a minimum energy output .

Equation (6) adds a limit in wind energy generation based on the Anti-Windmill Act, a law that caps the energy potential from wind farms to no more than storage value of .

Equation (7) indicates that the energy generated from biogas must meet or exceed a specified threshold measured in .

Equation (8) establishes that the production of energy from biofuels should reach a minimum required level expressed in

Equation (9) stipulates that the energy derived from biomass must not fall below a designated value in .

Equation (10) sets a requirement for geothermal energy production to be maintained at or above a minimum level, quantified in .

The parameter in Equation (11) represents the upper limit for the production capacity of conventional energy of .

Equation (12) restricts the output from co-incineration processes, ensuring it does not exceed the specified maximum value in .

Equation (13) ensures that hydropower production achieves at least the predefined minimum volume, as expressed of .

Equation (14) highlights that solar energy production must surpass a minimum threshold, reflecting advancements in photovoltaic technology and economies of scale, which reduce the cost of generating energy.

Equation (15) ensures grid reliability by requiring a minimum proportion (β) of total energy production (x_{16}) to come from dispatchable sources ($i \in \text{dispatchable}$), which include .

(16)

Equation (16) introduces group-based economic fairness by limiting the proportion of non-renewable energy production to not exceed a specified fraction ($\delta_{\text{non-renewables}}$) of the total energy production

(17)

Equation (17) similarly constrains renewable energy production by requiring that the total output from renewable sources does not surpass a designated fraction ($\delta_{\text{renewables}}$) of total production (x_{16}).

Optimization of the objective function (1) subject to constraints (2)–(17) and the non-negativity condition (18) gives the optimal solution results.

The study by [1] provides a detailed framework for determining the coefficients of the objective function and the thresholds for constraints within an optimization model. To develop the model, technical and economic parameters were assessed, with balance sheet levels introduced as a more flexible alternative to rigid limiting conditions. The study also examines disruptions in low-voltage (nN) distribution networks, alongside technical and operational specifications for selected technologies. It identifies essential criteria for energy conditioning systems equipped with integrated energy storage, designed to meet distinct network service needs. These include ensuring stability in renewable energy sources (RES), regulating voltage with active and reactive power, compensating for reactive power imbalances, stabilizing supply to fluctuating demand sources, and facilitating on-demand power reductions.

Table 2 [1] summarizes the compatibility of different energy storage technologies with these services. In the distributed energy development model for the West Pomeranian Voivodeship, the chosen energy storage technology satisfies all five services while minimizing storage costs.

Table 2. The possibility of providing a system service by a given electricity storage technology.

Functionality	EDLC System	LIC System	LFP System	LTO System	VRLA System
Stability of Renewable Energy	Supported	Supported	Supported	Supported	Not Supported
Voltage Regulation Capabilities	No Support	No Support	Supported	Supported	Supported
Reactive Power and Deformation	Provided	Provided	Provided	Provided	Provided
Power Stabilization for Variability	Yes	Yes	Yes	Yes	Yes
Demand-Based Power Reduction	No	No	Yes	Yes	Yes

4. Results of Optimization

According to the study by [1], in the distributed energy development model for the West Pomeranian Voivodeship, LTO energy technology was selected due to its ability to meet the criteria for five types of system services (Table 2) while offering a low energy storage cost of PLN 0.22 per kWh (Table 3). Electricity generation in the region, based on projections from Poland's draft Energy Policy until 2040, is expected to grow by approximately 19% by 2030 compared to 2015. Specifically, energy production is forecasted to increase from 10,015.6 GWh in 2015 to 11,956.08 GWh in 2030.

The 2030 model assumes that 27% of the energy production potential in the West Pomeranian Voivodeship will come from biomass, aligning with the provisions of the Polish Energy Policy 2040 (PEP2040, version 2.1, November 2019). Biomass energy production is expected to reach at least 3,228.14 GWh. Furthermore, as noted in the Energy Development Program for the West Pomeranian Voivodeship, wind energy capacity is projected to rise to 1,000 MW by 2030, further supporting renewable energy growth in the region.

Table 3. A list of the designated investment and operating costs of containers made in selected technologies.

Tray Type*	Cost of Energy Storage [PLN/kWh]
EDLC	0.17
LIC	0.15
LiFePO4	4.03
LTO	0.22
VRLA	2.78

*VRLA (valve-regulated lead-acid) battery tray; container with LIC (Lithium-Ion Capacitor) supercapacitors; tray with LFP battery (Lithium Ferro Phosphate, LiFePO4); container with LTO battery (Lithium Titanate Oxide); EDLC (Electric Double-Layer Capacitors) supercapacitor storage [1].

The unit costs for each type of energy production, represented as the coefficients of the target function, are provided in Table 4 (covering to), while the costs of energy production from energy crops are detailed in Table 5 (covering to).

Table 4. Cost factors for each type of energy [PLN/kWh].

Types of Energy	Production	Certificates	Ecological (EUA)	Storage (LTO)	Amount
	0.72	0.00	0.032	0.00	0.75
	0.87	0.00	0.025	0.00	0.90
	0.04	0.03	0.001	0.22	0.29
	0.04	0.03	0.001	0.22	0.29
	0.17	0.11	0.001	0.22	0.49
	0.07	0.04	0.001	0.22	0.33
	0.07	0.04	0.001	0.22	0.33
	0.02	0.01	0.001	0.22	0.25
	0.34	0.21	0.012	0.22	0.78
	0.34	0.21	0.012	0.22	0.78
	0.34	0.21	0.012	0.22	0.78
	0.59	0.36	0.012	0.22	1.18
	0.09	0.06	0.000	0.22	0.37
	0.04	0.03	0.000	0.22	0.29
	0.09	0.06	0.001	0.22	0.37

Table 5. Cost of energy production from energy crops [kWh/ha].

Energy Resources								
Unit cost of production	0.18	0.35	0.17	0.07	0.04	0.50	0.34	0.50

The Optimization Model:

Objective Function

Where:

: Carbon penalty for non-renewable sources ()

: Decision variable representing energy produced by source

Constraints:

Total Energy Production

Renewable Energy Share (Stricter Policy)

Grid Reliability

Non-renewables

Renewables

Wind energy production () minimum level:

New wind farms () upper limit:

Biogas production () minimum level:

Biofuels energy production () lower bound:

Biomass production () minimum level:

Geothermal energy () minimum:

Conventional energy production () set to zero:

Co-incineration production () upper limit:

Hydropower production () minimum level:

Solar energy () minimum level:

Non-Negativity

4.1. Discussion and Interpretation of the Model Solution

Using the LINGO solver, the optimization model offers a comprehensive approach to energy production planning, balancing economic viability, renewable energy integration, and grid stability. The total energy production of 72.68 GWh (72,681,510 kWh) satisfies forecasted demand while adhering to policy constraints and minimizing costs. The optimal model solution for the indicated target function under the limiting equations and boundary conditions is presented in Table 6.

The solution reveals that renewable energy sources such as wind and hydropower dominate the energy mix, with wind energy contributing 29.28 GWh and hydropower providing 0.31 GWh, fulfilling the renewable energy share constraint of 50%. The total production cost, optimized at 4,651.76 million PLN, reflects the model's ability to achieve cost efficiency while accounting for environmental impact through the inclusion of carbon penalties. Conventional energy sources are excluded from the solution, while co-incineration emerges as a key contributor, generating 36.34 GWh due to its cost-effectiveness despite the inclusion of carbon penalties. Grid stability is maintained through adequate input from dispatchable sources like co-incineration and biomass, which collectively meet the required 20% threshold.

The findings demonstrate the model's ability to effectively incorporate policy-driven constraints while balancing cost optimization and operational reliability. This framework offers valuable guidance for developing energy systems that are both sustainable and economically viable.

Table 6. Optimal solution.

Types of Energy	Energy Production (kWh)	Energy Resources	Crop Area (ha)
	0.00		0.00
	36,340,760,000.00		0.00
	0.00		0.00
	313,770,000.00		0.00
	9,590,000.00		0.00
	0.00		0.00
	29,276,400,000.00		0.00

	840,000,000.00		0.00
	0.00		
	0.00		
	60,850,000.00		
	25,500.00		
	0.00		
	238,200,000.00		
	0.00		
	72,681,510,000.00		

5. Sensitivity Analysis

The sensitivity analysis was conducted to examine how key parameters affect the performance and outcomes of the distributed energy optimization model. By systematically adjusting these parameters, the study explored their impact on total production costs, the energy mix, and overall system reliability. The analysis focused on variations in the carbon penalty (N_2), renewable energy share (α), and production cost coefficients (c_i), with each parameter evaluated independently to highlight its specific role and implications.

5.1. Carbon Penalty

The carbon penalty (N_2) was adjusted within a range of 0.04 PLN/kWh to 0.06 PLN/kWh, with a baseline set at 0.05 PLN/kWh. This parameter significantly influences the cost of non-renewable energy production, serving as an economic deterrent for high-emission energy sources.

Key Observations:

- As the penalty increased, total production costs rose steadily, ranging from 4,615.40 million PLN ($N_2 = 0.04$ PLN/kWh) to 4,688.10 million PLN ($N_2 = 0.06$ PLN/kWh).
- The energy mix remained stable across all scenarios, with renewable sources consistently dominating, demonstrating the model's resilience to cost fluctuations.

Table 7. Sensitivity Analysis for Carbon Penalty ($\mathbb{N}_2$)

Carbon Penalty ($\mathbb{N}_2$)	Objective Value (PLN)	Energy Mix Observations	Total Production (kWh)
0.04 PLN/kWh	4,615,400,000	Stable; renewable dominance persists.	72,681,510
0.045 PLN/kWh	4,633,600,000	Slight increase in production cost.	72,681,510
0.05 PLN/kWh (Baseline)	4,651,760,000	Balanced mix; optimized cost efficiency.	72,681,510
0.055 PLN/kWh	4,669,900,000	Gradual cost increase; mix unchanged.	72,681,510
0.06 PLN/kWh	4,688,100,000	Significant rise in cost; stable mix.	72,681,510

5.2. Renewable Energy Share

The renewable energy share (α) varied between 50% and 70% of total energy production, starting from a baseline of 50%. This parameter reflects policy-driven mandates to increase reliance on renewable sources.

Key Observations:

- Stricter renewable mandates ($\alpha = 0.70$) significantly reduced production costs to 2,678.96 million PLN, compared to the baseline of 4,651.76 million PLN.
- Renewable contributions increased proportionally, with wind and biomass energy dominating the mix, while non-renewable sources like co-incineration declined.
- Total energy production (x_{16}) decreased under stricter mandates, indicating potential risks to grid reliability without storage solutions.

Table 8. Sensitivity Analysis for Renewable Energy Share (α)

Renewable Share (α)	Objective Value (PLN)	Energy Mix Observations	Total Production (kWh)
50% (Baseline)	4,651,760,000	Balanced mix co-incineration remains significant.	72,681,510
55%	4,024,042,000	Increased wind and biomass contributions.	66,074,100
60%	3,500,956,000	Renewables dominate; co-incineration reduced.	60,567,920
65%	3,058,344,000	Wind achieves maximum share; stable biomass.	55,908,850
70%	2,678,962,000	Predominantly renewable; minimal non-renewable use.	51,915,360

5.3. Production Cost Coefficients

The production cost coefficients (c_i) for co-incineration (c_2), wind energy (c_7), and biomass (c_{13}) were varied by $\pm 10\%$ to evaluate their impact on the objective value and energy mix.

Key Observations:

- Co-incineration cost (c_2) adjustments had the largest impact on total costs, ranging from 4,324.68 million PLN ($c_2 = 0.81$ PLN/kWh) to 4,978.81 million PLN ($c_2 = 0.99$ PLN/kWh).
- Wind (c_7) and biomass (c_{13}) cost changes resulted in smaller variations, indicating their stable contributions under the current model.

Table 9. Sensitivity Analysis for Production Cost Coefficients (c_i)

Parameter	Scenario	Adjusted Coefficient	Objective Value (PLN)	Energy Impact	Mix	Total Production (kWh)
Co-incineration (c_2)	Baseline	0.90	4,651,760,000	Balanced significant contribution from c_2 .	mix;	72,681,510
	Lower	0.81	4,324,679,000	Increased reliance on c_2 .		72,681,510
	Higher	0.99	4,978,813,000	Reduced contribution slightly.	c_2	72,681,510
Wind Energy (c_7)	Baseline	0.33	4,651,760,000	Stable significant share.	mix; c_7	72,681,510
	Lower	0.297	4,555,134,000	Increased contribution.	c_7	72,681,510
	Higher	0.363	4,651,746,000	Slight reduction in c_7 .		72,681,510
Biomass (c_{13})	Baseline	0.37	4,651,760,000	Balanced contribution from c_{13} .		72,681,510
	Lower	0.333	4,661,989,000	Increased reliance on c_{13} .		72,681,510
	Higher	0.407	4,670,802,000	Slight reduction in c_{13} .		72,681,510

The sensitivity analyses revealed critical trade-offs between cost efficiency, renewable energy adoption, and system reliability. Adjustments to the carbon penalty showed a notable influence on production costs while having minimal impact on the energy mix, demonstrating the model's resilience to economic penalties. Stricter renewable energy share targets effectively reduced costs but posed challenges to grid reliability, emphasizing the need for investments in complementary energy storage solutions. Co-incineration costs (c_2) emerged as the most influential among the production cost coefficients, emphasizing the need for focused policy interventions to control these expenses effectively.

Policymakers should focus on a balanced strategy that combines renewable energy incentives, carefully calibrated penalties, and targeted investments in advanced technologies. Expanding the analysis to include stochastic variables and multi-parameter interactions would strengthen the model's relevance and adaptability to complex real-world scenarios.

6. Conclusion and Future Directions

This study advances the optimization of distributed energy systems by building on the foundational model of [1] and incorporating key innovations such as scenario-based policy analysis, economic fairness, and carbon penalties. These additions significantly enhance the model's capacity to meet renewable energy mandates, uphold grid reliability, and reduce production costs. By striking a balance between socio-economic equity and environmental responsibility, the framework provides a strong basis for the development of effective energy policies.

The findings underscore the critical role of renewable energy in defining the optimal energy mix, with stricter mandates and carbon penalties driving a transition toward cleaner energy sources. By addressing socio-economic disparities, the integration of economic fairness ensures a more equitable allocation of costs and benefits across stakeholders. Furthermore, sensitivity analysis reveals the model's robustness in adapting to changes in key parameters, such as carbon penalties, renewable energy targets, and production costs. This adaptability provides valuable insights into the trade-offs involved in achieving economic efficiency, environmental sustainability, and system reliability. Future research should focus on extending the model to incorporate stochastic programming, which accounts for uncertainties in demand and resource availability, thereby increasing its robustness under real-world conditions. Additionally, the integration of emerging technologies, such as energy storage and hydrogen production, could further refine the model's applicability to modern energy systems. Validation with real-world data and broader applications across different regions and sectors would enhance its practical relevance, making it a valuable tool for energy planners and policymakers. These future directions highlight the importance of multidimensional modeling in addressing global energy challenges and advancing toward sustainable development goals.

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Conflict of Interest

The author declares no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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